

Further Mathematics Paper 2, May/June. 2012

Question 1

Two functions g and h are defined on the set R of real numbers by
 $g: x \rightarrow x^2 - 2$ and $h: x \rightarrow \frac{1}{x+2}, x \neq -2$.

Find:

- (a) h^{-1} , the inverse of h ;
- (b) $g \circ h$ when $x = -\frac{1}{2}$

Question 2

Write down the first three terms of the binomial expansion $(1 + ax)^n$ in ascending powers of x . If the coefficients of x and x^2 are 2 and $\frac{3}{2}$ respectively, find the values of a and n .

Question 3

Express $3x^2 - 6x + 10$ in the form $a(x - b)^2 + c$ where a , b , and c are integers. Hence, state the minimum value of $3x^2 - 6x + 10$ and the value of x for which it occurs.

Question 4

The twenty-first term of an Arithmetic Progression (AP) is $5\frac{1}{2}$ and the sum of the first twenty-one terms is $94\frac{1}{2}$.

Find the:

- (a) first term;
- (b) common difference;
- (c) sum of the first thirty terms.

Question 5

The gradient function of $y = ax^2 + bx + c$ is $8x + 4$. If the function has a minimum value of 1, find the values of a, b and c.

Question 6

Three forces $-63\mathbf{j}$, $32.14\mathbf{i} + 38.3\mathbf{j}$ and $14\mathbf{i} - 24.25\mathbf{j}$ act on a body of mass 5 kg.

Find, correct to one decimal place, the:

- (a) magnitude of the resultant force;
- (b) acceleration of the body.

Question 7

Simplify: ${}^{n+1}C_4 - {}^{n-1}C_4$

Question 8

The marks scored by 35 students in a test are given in the table below:

Marks	1 - 5	6 - 10	11 - 20	21 - 30	31 - 35	36 - 40
No of students	2	7	12	8	5	1

Draw a histogram for the distribution.

Question 8

The marks scored by 35 students in a test are given in the table below:

Marks	1 - 5	6 - 10	11 - 20	21 - 30	31 - 35	36 - 40
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Draw a histogram for the distribution.

Question 9

(a) The polynomial $f(x) = x^3 + px^2 - 10x + q$ is exactly divisible by $(x^2 + x - 6)$. Find the:

1. values of p and q
2. third factor.

(b) The volume of a cube is increasing at the rate of $2\frac{1}{2} \text{ cm}^3\text{s}^{-1}$. Find the rate of change of the side of the cube when its length is 2 cm.

Question 10

(a) Write down the matrix \mathbf{A} of the linear transformation
 $A(x, y) \rightarrow (2x - y, -5x + 3y)$.

(b) If $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$, find:

- (i) $\mathbf{A}^2 - \mathbf{B}^2$;
- (ii) matrix $\mathbf{C} = \mathbf{B}^2\mathbf{A}$;
- (iii) The point $M(x, y)$ whose image under the linear transformation \mathbf{C} is $M(10, 18)$.

(c) What is the relationship between matrix \mathbf{A} and matrix \mathbf{C} ?

Question 11

(a) Evaluate: $\int_1^4 \frac{x(3x-2)}{2\sqrt{x}} dx$.

(b) The equation of a circle is given by $2x^2 + 2y^2 - 8x + 5y - 10 = 0$. Find the:
(i) coordinates of the centre;
(ii) radius of the circle;
(iv) Coordinates of P and Q, if the circle cuts the x-axis at the points P and Q.

Question 12

(a) (i) Find the sum of the series

$$A(1+r) + A(1+r)^2 + \dots + A(1+r)^n.$$

(ii) Given that $r = 8\%$ and $A = \text{GH}\text{¢ } 40.00$, find the sum of the 6th to 10th terms of the series in 12(a) (i).

(b) Find the equation of the tangent to the curve $y = \frac{1}{x}$ at the point on the curve when $x = 2$.

Question 13

- (a) A fair die with six faces is thrown six times. Calculate, correct to three decimal places, the probability of obtaining:
- (i) exactly three sixes;
 - (ii) at most three sixes.
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- (b) Eight percent of screws produced by a machine are defective. From a random sample of 10 screws produced by the machine, find the probability that:
- (i) exactly two will be defective;
 - (ii) not more than two will be defective.
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Question 14

The table gives the distribution of heights in metres of 100 students

Height	1.40 -1.42	1.43-1.45	1.46-1.48	1.49-1.51	1.52-1.54	1.55-1.57	1.58-1.60	1.61-1.63
Frequency	2	4	19	30	24	14	6	1

- (a) Calculate the:
- mean height;
 - mean deviation of the distribution.
- (b) What is the probability that the height of a student selected at random is greater than the mean height of the distribution?

Question 15

(a) Two items are selected at random from four items labelled (p, q, r, s).

(i) List the sample space if sampling is done:

- with replacement;
- without replacement.

- Find the probability that r is at least one of the two objects selected:

- (b) in a(i) μ ;
- (c) in a (i) b.

(b) How many whole numbers from 100 to 999 are divisible by:

- 4;
- both 3 and 4?

Question 16

(a) A body P of mass 5 kg is suspended by two light inextensible strings AP and BP attached to a ceiling. If the strings are inclined at angles 40° and 30° respectively to the downward vertical, find the tension in each of the strings. {Take $g = 10\text{ms}^{-2}$ }.

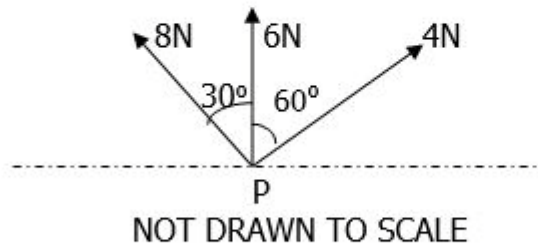
(b) A constant force F acts on a toy car of mass 5 kg and increases its velocity from 5 ms^{-1} to 9 ms^{-1} in 2 seconds. Calculate the:

- magnitude of the force;
- velocity of the toy car 3 seconds after attaining a velocity of 9 ms^{-1} .

Question 17

- (a) Given that $\mathbf{p} = (4\mathbf{i} - 3\mathbf{j})$ and $\mathbf{q} = (-\mathbf{i} + 5\mathbf{j})$, find \mathbf{r} such that $|\mathbf{r}| = 15$ and is in the direction of $(2\mathbf{p} + 3\mathbf{q})$.

(b)



Forces of magnitude $8N$, $6N$ and $4N$ act at the point P , as shown in the diagram above. Find the:

- (i) magnitude;
- (ii) direction of the resultant force.

Question 18

(a) Find the angle between the vectors $\mathbf{a} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -8 \\ -15 \end{pmatrix}$.

(b) Given that $\mathbf{a} = (4 \text{ N}, 060^\circ)$ and $\mathbf{b} = (3 \text{ N}, 120^\circ)$, find, in component form, the unit vector along $\mathbf{a} - \mathbf{b}$.